Probabilistic Sharpe Ratio Optimization

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Abstract

This vignette gives an overview of the Probabilistic Sharpe Ratio , Minimum Track Record Length and the Probabilistic Sharpe Ratio Optimization technique used to find the optimal portfolio that maximizes the Probabilistic Sharpe Ratio. It gives an overview of the usability of the functions and its application.

A probabilistic translation of Sharpe ratio, called PSR, is proposed to account for estimation errors in an IID non-Normal framework.When assessing Sharpe ratioâĂŹs ability to evaluate skill,we find that a longer track record may be able to compensate for certain statistical shortcomings of the returns probability distribution. Stated differently, despite Sharpe ratio's well-documented deficiencies, it can still provide evidence of investment skill, as long as the user learns to require the proper track record length.

The portfolio of hedge fund indices that maximizes Sharpe ratio can be very different from the portfolio that delivers the highest PSR. Maximizing for PSR leads to better diversified and more balanced hedge fund allocations compared to the concentrated outcomes of Sharpe ratio maximization.

1 Probabilistic Sharpe Ratio

Given a predefined benchmark Sharpe ratio SR^* , the observed Sharpe ratio $\hat{SR}$ can be expressed in probabilistic terms as

$$P\hat{S}R(SR^*) = Z\Big[\frac{(\hat{SR} - SR^*)\sqrt{n-1}}{\sqrt{1 - \hat{\gamma_3}SR^* + \frac{\hat{\gamma_4}-1}{4}\hat{SR}^2}}\Big]$$

Here n is the track record length or the number of data points. It can be daily,weekly or yearly depending on the input given

$\hat{\gamma_3}$nd $\hat{\gamma_4}$re the skewness and kurtosis respectively. It is not unusual to find strategies with irregular trading frequencies, such as weekly strategies that may not trade for a month. This poses a problem when computing an annualized Sharpe ratio, and there is no consensus as how skill should be measured in the context of irregular bets. Because PSR measures skill in probabilistic terms, it is invariant to calendar conventions. All calculations are done in the original frequency of the data, and there is no annualization. The Reference Sharpe Ratio is also given in the non-annualized form and should be greater than the Observed Sharpe Ratio.

```
> data(edhec)
> ProbSharpeRatio(edhec[,1],refSR = 0.23)

                                          Convertible Arbitrage (SR > 0.23 )
Probabilistic Sharpe Ratio(p= 95 %):                               0.7689223
```

2 Minimum Track Record Length

If a track record is shorter than Minimum Track Record Length(MinTRL), we do not have enough confidence that the observed $\hat{SR}$s above the designated threshold SR^* Minimum Track Record Length is given by the following expression.

$$MinTRL = n^* = 1 + \left[1 - \hat{\gamma_3}\hat{SR} + \frac{\hat{\gamma_4}}{4}\hat{SR^2}\right]\left(\frac{Z_\alpha}{\hat{SR} - SR^*}\right)^2$$

γ_3nd $\gamma_4$re the skewness and kurtosis respectively. It is important to note that MinTRL is expressed in terms of number of observations, not annual or calendar terms. All the values used in the above formula are non-annualized, in the same frequency as that of the returns.

```
> data(edhec)
> MinTrackRecord(edhec[,1],refSR = 0.23)

                                          Convertible Arbitrage (SR > 0.23 )
Probabilistic Sharpe Ratio(p= 95 %):                                 756.613
```

3 Probabilistic Sharpe Ratio Optimal Portfolio

We would like to find the vector of weights that maximize the expression

$$P\hat{S}R(SR^*) = Z\left[\frac{(\hat{SR} - SR^*)\sqrt{n-1}}{\sqrt{1 - \hat{\gamma_3}SR^* + \frac{\hat{\gamma_4}-1}{4}\hat{SR^2}}}\right]$$

where $\sigma = \sqrt{E[(r-\mu)^2]}$its standard deviation.$\gamma_3 = \frac{E\left[(r-\mu)^3\right]}{\sigma^3}$ts skewness,$\gamma_4 = \frac{E\left[(r-\mu)^4\right]}{\sigma^4}$ts kurtosis and $SR = \frac{\mu}{\sigma}$ts Sharpe Ratio.

Because $P\hat{S}R(SR^*) = Z[\hat{Z}^*]$s a monotonic increasing function of $\hat{Z}^*$it suffices to compute the vector that maximizes $\hat{Z}^*$his optimal vector is invariant of the value adopted by the parameter SR^*

```
> data(edhec)
> PsrPortfolio(edhec)
```

```
                              weight
Convertible Arbitrage   0.0006284846
CTA Global              0.0724237635
Distressed Securities   0.0724237635
Emerging Markets        0.0724237635
Equity Market Neutral   0.0724237635
Event Driven            0.0724237635
Fixed Income Arbitrage  0.0724237635
Global Macro            0.0724237635
Long/Short Equity       0.0724237635
Merger Arbitrage        0.0724237635
Relative Value          0.0724237635
Short Selling           0.2027101166
Funds of Funds          0.0724237635
```